

[1] If  $F(x, y, z) = x^3 e^z + y \sin z - z \cos y$

Find: (a) The first derivatives of the function  $F$  (b)  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  (c)  $\nabla F$  and  $\nabla \cdot \nabla F$

[2] Show that the envelope of the curves:  $(x - 2a)^2 + (y - 2a)^2 = 1$  is  $(x - y)^2 = 4$ .

[3] Determine the extrema of the function:  $f(x, y) = x^3 + y^2 - 3x - 4y$

[4] From the curve:  $x = t^3 + t$ ,  $y = t^2 + 3$ ,  $t$  in  $[1, 2]$ .

Find the area  $A$ , the length  $L$  and the volume  $V_x$ .

[5] Find  $\int_{(0,1)}^{(2,5)} (x^2 + y)dx + (xy)dy$  through the curve  $y = x^2 + 1$

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*Good Luck*

*Dr. Mohamed Eid*

[1] If  $F(x, y, z) = y^3 e^x + x \cos z - z^2 \sin y$

Find: (a) The first derivatives of the function  $F$  (b)  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  (c)  $\nabla F$  and  $\nabla \cdot \nabla F$

[2] Find the envelope of the curves:  $x \cos a + y \sin a = 1$ .

[3] Determine the extrema of the function:  $f(x, y) = x^3 + (y + 2)^2 - 27x$

[4] From the curve:  $x = e^t, y = t^3, t$  in  $[0, 2]$ .

Find the area  $A$ , the length  $L$  and the volume  $V_x$ .

[5] Find  $\int_{(0,1)}^{(1,2)} (x + y)dx + (2x - y)dy$  through the curve  $y = x^3 + 1$

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[1] If  $F(x, y, z) = z^3 e^y + y \cos y - x^2 \sin z$

Find: (a) The first derivatives of the function  $F$  (b)  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  (c)  $\nabla F$  and  $\nabla \cdot \nabla F$

[2] Show that the envelope of the curves:  $(x - 2a)^2 + (y + 2a)^2 = 1$  is  $(x + y)^2 = 4$ .

[3] Determine the extrema of the function:  $f(x, y) = (x - 3)^2 + y^3 - 12y$

[4] From the curve:  $x = t^3, y = t^3 + 2, t$  in  $[1, 3]$ .

Find the area  $A$ , the length  $L$  and the volume  $V_x$ .

[5] Find  $\int_{(0,1)}^{(1,3)} (x + y^2)dx + (x^2 y)dy$  through the curve  $y = 2x^2 + 1$

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[1] If  $F(x, y, z) = y^3 e^{2x} - x \sin y + z \cos z$

Find: (a) The first derivatives of the function  $F$  (b)  $\frac{\partial z}{\partial y}$  (c)  $\nabla F$  and  $\nabla \cdot \nabla F$

[2] Find the envelope of the curves:  $(x - a)^2 + (y + 2a)^2 = 1$ .

[3] Determine the extrema of the function  $f(x, y) = x^2 + y^3 - 4xy + 4y$

[4] Determine the extrema of the function  $f(x, y) = x^3 + y^2$  s.t  $3x + 2y = 9$

[5] From the curve :  $x = t^3$ ,  $y = 3 + e^t$ ,  $t$  in  $[1, 2]$ .

Find the area  $A$ , the length  $L$  and the volume  $V_y$ .

[6] Find  $\int_{(1,0)}^{(2,1)} (x^2 + y)dx + (y - x)dy$  through the curve  $x = 1 + y^2$

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(1) If  $f(z) = z + e^z$ . Show that  $u(x, y)$ ,  $v(x, y)$  satisfy the Riemann's equations.

(2) If  $u = x + e^x \cos y$ . Find its conjugate  $v(x, y)$  such that  $f(z) = u + iv$  is harmonic.

(3) Determine and sketch the image of the region  $G : 0 \leq y \leq 2, -\pi/2 \leq x \leq \pi/2$

under the function  $f(z) = \sin z$ .

(4) If  $C$  is the circle  $|z - 1| = 3$ . Find the integrals:

$$(a) \oint_C \frac{\sin z}{z^2 + 9} dz \quad (b) \oint_C \frac{\cos z}{(2z - \pi)^2} dz \quad (c) \oint_C \frac{3^z}{(z - 1)(z - 2)} dz$$

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- (1) If  $f(z) = 3 + \ln z$ . Show that  $u(x, y)$ ,  $v(x, y)$  satisfy the Riemann's equations.
- (2) If  $u = 3 + e^{2x} \sin 2y$ . Find its conjugate  $v(x, y)$  such that  $f(z) = u + iv$  is harmonic.
- (3) Determine and sketch the image of the region  $G : \ln 3 \leq x \leq \ln 4, 0 \leq y \leq \pi$  under the function  $f(z) = e^z$ .
- (4) If  $C$  is the circle  $|z - 2i| = 2$ . Find the integrals:

$$(a) \oint_C \frac{3^z}{z^2 - 9} dz \quad (b) \oint_C \frac{\cos z}{2z - \pi i} dz \quad (c) \oint_C \frac{2^{iz}}{(z - i)(z - 3i)} dz$$

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- (1) If  $f(z) = z + \cos z$ . Show that  $u(x, y)$ ,  $v(x, y)$  satisfy the Riemann's equations.
- (2) If  $v = \sin x \cdot \sinh y$ . Find its conjugate  $u(x, y)$  such that  $f(z) = u + iv$  is harmonic.
- (3) Determine and sketch the image of the region  $G : 0 \leq y \leq 3, 0 \leq x \leq \frac{\pi}{2}$  under the function  $f(z) = \sin z$ .
- (4) If  $C$  is the circle  $|z + 3| = 2$ . Find the integrals:

$$(a) \oint_C \frac{3^z}{z^3 - 36z} dz \quad (b) \oint_C \frac{\cos z}{z + \pi} dz \quad (c) \oint_C \frac{z^2}{(z + 2)(z + 4)} dz$$

*Good Luck*

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